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Tight-binding description of single-mode cavity-plasmon waveguides in the frequency and time domain

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Abstract
We report a consistent derivation of a tight-binding formalism, both in the frequency and in the time domain, for the analysis of electromagnetic energy transfer in single-mode cavity-plasmon waveguides. Moreover, we derive closed-form solutions of the relevant tight-binding equations, which describe the response of these waveguides under time-varying excitations by a localized light source. In this context, we discuss the possibility of efficient single-mode waveguiding through coupled cavity-plasmon modes in chains of spheroidal silicon nanoparticles in silver at optical frequencies.

1. Introduction

We have recently reported on light propagation through a chain of equally spaced identical dielectric nanoparticles in a metallic host medium by a hopping mechanism between localized cavity plasmons [1]. The purpose of the present paper is to propose and analyze a specific design of this so-called cavity-plasmon waveguide, consisting of spheroidal silicon nanoparticles in silver, which ensures single-mode operation at visible frequencies and can be realized in the laboratory using modern nanofabrication techniques. This waveguide represents a means of light localization and transport at subwavelength spatial dimensions, and thus may enable the fabrication of integrated nanoscale optical components. On the other hand, single-mode cavity-plasmon waveguides constitute a class of actual photonic structures that can be accurately described using simple tight-binding models, introduced in the first instance in relation to electron states in solids [2]. This allows one to study a variety of interesting physical phenomena, including effects associated with the presence of defects, disorder and nonlinearity, in a simple and straightforward manner, enabling physical insight. Electromagnetic (EM) tight-binding formalisms for chains of weakly coupled cavities have been developed, in the frequency domain, within the framework of scattering theory [3] as well as following the traditional eigenmode-expansion approach based on the second-order wave equation for the electric component of the EM field [4]. In this paper, we present a consistent derivation of a tight-binding formalism for single-mode cavity-plasmon waveguides, both in the frequency and in the time domain, starting from the set of the coupled first-order Maxwell equations. Moreover, we derive spatio-temporal solutions, which describe the response of these waveguides under time-varying excitations by a localized light source, in a closed form.

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2. Design of single-mode cavity-plasmon waveguides

A single dielectric cavity of dielectric constant ϵ_s and magnetic permeability μ_s, in a metal, supports bound states of the EM field, at the poles of the scattering T matrix. Let us assume, to begin with, that the optical response of the metal is described by the simple yet effective Drude model dielectric function [2]

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\tau^{-1})}, \tag{1}$$

where ω_p is the bulk plasma frequency and τ is the relaxation time of the conduction-band electrons, and $\mu = 1$. For a spherical cavity of radius S, it can be shown that, below ω_p, poles of the T matrix exist on the real frequency axis, if absorptive losses are neglected, only for $T_{E\ell}(\omega)$, i.e. they correspond to electric multipole modes. For a small cavity, the

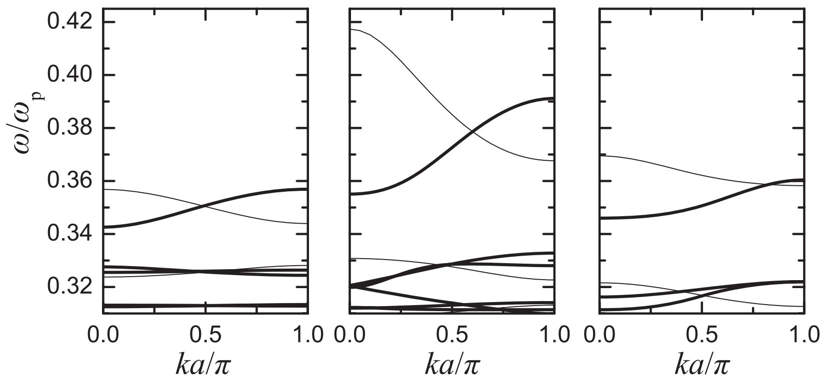

Figure 1. Frequency bands of linear periodic chains of dielectric nanospheres ($\epsilon_s = 12$, $\mu_s = 1$) in a non-absorbing metal, described by equation (1) with $\tau^{-1} = 0$. Left-hand diagram: $S = 0.5c/\omega_p$, $a = 2c/\omega_p$; middle diagram: $S = 0.5c/\omega_p$, $a = 1.2c/\omega_p$; right-hand diagram: $S = 0.85c/\omega_p$, $a = 2c/\omega_p$. The thick and thin lines show the doubly degenerate and non-degenerate bands, respectively.

eigenfrequencies of these so-called cavity-plasmon (because they are associated with 2^ℓ-pole plasma oscillations at the surface of the cavity) modes are given by $\overline{\omega}_\ell \simeq \omega_p[(\ell + 1)/(\ell\epsilon_s + \ell + 1)]^{1/2}$, $\ell = 1, 2, 3, \ldots$. In an infinite linear chain of such cavities, with their centers separated by a distance a, the plasmon modes of the individual cavities, of given ℓ, interact weakly between them and form narrow bands about $\overline{\omega}_\ell$. For chains of nanovoids in a Drude metal, these bands extend over a relatively narrow frequency region of about $0.8\omega_p$ [1], i.e. at ultraviolet frequencies. In the present paper, we consider nanocavities made of a high-dielectric-constant material, in order to push the waveguide bands down to optical and infrared frequencies. One of these bands (that which corresponds to $m = 0$) is non-degenerate. The rest of them, which correspond to $m = \pm1, \pm2, \ldots, \pm\ell$, are doubly degenerate. For relatively large values of a, the interaction between the cavities is weak and the different 2^ℓ-pole bands are very narrow and well separated from each other (see the left-hand diagram of figure 1). As a becomes smaller, the width of the bands increases, and the interaction between bands of the same symmetry is clearly manifested, as shown in the middle diagram of figure 1. Moreover, as the size of the cavities increases, the eigenfrequencies of the plasmon modes of the single cavity shift to lower frequencies and come closer to each other, which also leads to a stronger interaction between bands of the same symmetry, as shown in the right-hand diagram of figure 1. In any case, the bandwidth decreases with increasing ℓ because of the stronger localization of the higher 2^ℓ-pole cavity-plasmon modes.

For waveguide applications, the presence of a large number of interacting bands is undesirable. Therefore, we shall be concerned with situations where the dipole bands of the chain of cavities are well separated from the higher-multipole bands, and we shall study wave propagation at frequencies within the range of these dipole bands. At these frequencies, it is reasonable to take into account only the dipole contributions to the multipole expansions of the EM field. Moreover, the condition of having the dipole bands isolated from the higher multipole bands requires a weak interaction between the cavities which, in turn, allows one to assume only nearest-neighbor hopping. Making a Laurent (or Taylor) expansion about $\overline{\omega}_1$ of the relevant quantities within the relatively narrow frequency range of the dipole bands, we obtain the dispersion relations in the form [1]

$$\omega_\mu(k) = \overline{\omega}_1 + W_\mu \cos(ka), \tag{2}$$

with $\mu = \alpha, \beta$ referring to the non-degenerate and the doubly degenerate dipole bands, respectively. $W_\alpha = 6A[(\overline{q}a)^{-2} + (\overline{q}a)^{-3}]\exp(-\overline{q}a)$ and $W_\beta = -3A[(\overline{q}a)^{-1} + (\overline{q}a)^{-2} + (\overline{q}a)^{-3}]\exp(-\overline{q}a)$, where $\overline{q} \equiv \sqrt{-\epsilon(\overline{\omega}_1)}\overline{\omega}_1/c$ and $A \equiv \lim_{\omega\to\overline{\omega}_1}[(\omega - \overline{\omega}_1)T_{E1}(\omega)]$ are real quantities. Equation (2) describes reasonably well the dispersion curves of the narrow dipole bands of the cavity-plasmon waveguides under consideration. However, if the cavities are very close to each other and/or if their size is relatively large (the middle and right-hand diagrams of figure 1), the exact results deviate strongly from equation (2), as expected. In this case we obtain relatively wide bands and a maximum group velocity of $\sim 0.03c$.

One can design a single-mode cavity-plasmon waveguide, over a given frequency range, by deforming the spherical shape of the cavities. In this case the spherical symmetry of the single cavity is broken and the threefold degeneracy of the dipole plasmon mode is removed. For example, if instead of the above spherical cavities with radius $S = 0.5c/\omega_p$ we have prolate spheroidal cavities of the same volume, with the major axis (axis of revolution) $1.07c/\omega_p$ long and the minor axis $0.97c/\omega_p$ long, the threefold degenerate dipole mode of the single sphere at $0.350c/\omega_p$ splits into a non-degenerate mode at $0.369\omega_p$ and a doubly degenerate mode at $0.341\omega_p$ in the case of the spheroid. A linear periodic chain, with $a = 2c/\omega_p$, of such spheroidal cavities, with their major axis perpendicular to the chain, generates three non-degenerate bands about the eigenfrequencies of the dipole plasmon modes of the single cavity. Interestingly, in the frequency region around $0.369\omega_p$ there is only one non-degenerate band. The modes of this band are totally transmitted through the chain, however it bends from cavity to cavity on the plane normal to their major axis, provided that the distance between nearest neighbors remains the same [3].

The absorption of light by the constituent materials, which we have neglected so far, is of course the most serious shortcoming of the cavity-plasmon waveguide under

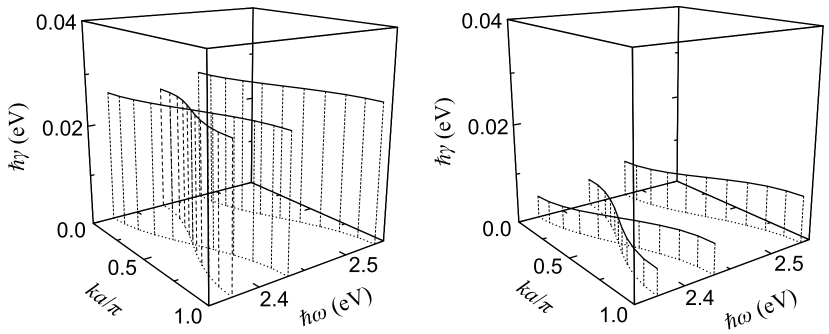

Figure 2. The dipole bands of a linear periodic chain of spheroidal silicon particles without gain ($\epsilon''_s = 0$, left-hand panel) and with gain ($\epsilon''_s = -0.6$, right-hand panel), in silver. The particles are separated by a distance of $a = 40$ nm; their major axis (axis of revolution, taken perpendicular to the chain) is 21.5 nm and their minor axis is 19.3 nm.

consideration. In the presence of losses, the poles of $T_{E\ell}$ are shifted off the real axis into the lower complex frequency half-plane at $z_\ell = \overline{\omega}_\ell - i\overline{\gamma}_\ell$, where $\overline{\gamma}_\ell > 0$ denotes the inverse of the lifetime of the respective cavity-plasmon mode, and the waveguide bands extend in the complex frequency plane, taking the form $\omega_\mu(k) - i\gamma_\mu(k)$. In the left-hand panel of figure 2 we show the dispersion diagrams in the complex frequency plane for a chain of spheroidal silicon nanoparticles in silver. For the dielectric functions of silicon and silver we interpolate to the experimental values given in [5] and [6], respectively. It can be seen that this waveguide ensures single-mode operation at visible frequencies (about 2.5 eV) and that $\gamma_\mu(k)$ does not vary appreciably with k, being roughly equal to the (negative) imaginary part of the corresponding eigenfrequency of the single silicon particle in the metal host.

We now explore the possibility of compensating for the losses by infiltrating the silicon particles with an optical gain material, which is capable of sustaining an inversion of the population under excitation by light of a different wavelength or by electric discharge. The resulting lasing activity can be well represented by a negative imaginary part ($\epsilon''_s < 0$) contributing to the dielectric constant of the silicon cavity, $\epsilon_s = \epsilon'_s + i\epsilon''_s$, whose value can be easily controlled. In the right-hand panel of figure 2 we show the dispersion diagrams in the complex frequency plane if we add an imaginary part ($\epsilon''_s = -0.6$) to the dielectric constant of silicon. It can be seen that the bands approach the real axis, which means that the waveguide modes become less dissipative. If we increase gain, the absorptive losses are further reduced.

3. Frequency- and time-domain tight-binding method: formalism and calculations

The dispersion relations of the cavity-plasmon waveguide, given by equation (2), can be obtained alternatively by an analysis similar to that used in the standard tight-binding method for electron states in solids [2]. Such an approach, based on the second-order wave equation for the electric component of the EM field, was followed by Yariv *et al* [4] in relation to coupled-cavity waveguides. Here we present a somewhat different derivation, starting from the coupled first-order Maxwell equations in a medium without free charges and currents, which we cast into the form

$$\begin{pmatrix} 0 & \frac{i}{\sqrt{\epsilon_0\epsilon(\mathbf{r})}}\nabla\times\frac{1}{\sqrt{\mu_0\mu(\mathbf{r})}} \\ \frac{-i}{\sqrt{\mu_0\mu(\mathbf{r})}}\nabla\times\frac{1}{\sqrt{\epsilon_0\epsilon(\mathbf{r})}} & 0 \end{pmatrix} \times \begin{pmatrix} \sqrt{\epsilon_0\epsilon(\mathbf{r})}\mathbf{E}(\mathbf{r},t) \\ \sqrt{\mu_0\mu(\mathbf{r})}\mathbf{H}(\mathbf{r},t) \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} \sqrt{\epsilon_0\epsilon(\mathbf{r})}\mathbf{E}(\mathbf{r},t) \\ \sqrt{\mu_0\mu(\mathbf{r})}\mathbf{H}(\mathbf{r},t) \end{pmatrix}. \quad (3)$$

The solutions of equation (3) must fulfill, in addition, the conditions $\nabla\cdot[\epsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t)] = 0$ and $\nabla\cdot[\mu(\mathbf{r})\mathbf{H}(\mathbf{r},t)] = 0$. The time-harmonic eigenmodes $\mathbf{E}(\mathbf{r},t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})\exp(-i\omega t)]$, $\mathbf{H}(\mathbf{r},t) = \mathrm{Re}[\mathbf{H}(\mathbf{r})\exp(-i\omega t)]$ are obtained from the eigenvalue equation $\widehat{\Theta}|F\rangle = \omega|F\rangle$, where the Maxwell operator $\widehat{\Theta}$ is represented by the matrix in the left-hand side of equation (3) and $\mathbf{F}(\mathbf{r}) = \begin{pmatrix} \sqrt{\epsilon_0\epsilon(\mathbf{r})}\mathbf{E}(\mathbf{r}) \\ \sqrt{\mu_0\mu(\mathbf{r})}\mathbf{H}(\mathbf{r}) \end{pmatrix}$ is a vector with six components $F_i(\mathbf{r}) \equiv \langle i\mathbf{r}|F\rangle$, $i = 1, 2, \ldots, 6$. Adopting the usual definition of the inner product

$$(F_1, F_2) = \int d^3r\big[\epsilon_0\epsilon(\mathbf{r})\mathbf{E}_1^*(\mathbf{r})\cdot\mathbf{E}_2(\mathbf{r}) + \mu_0\mu(\mathbf{r})\mathbf{H}_1^*(\mathbf{r})\cdot\mathbf{H}_2(\mathbf{r})\big], \quad (4)$$

it is straightforward to show that $(F_1, \widehat{\Theta}F_2) = (\widehat{\Theta}F_1, F_2)$, i.e. that $\widehat{\Theta}$ is a Hermitian operator, provided that $\epsilon(\mathbf{r})$ and $\mu(\mathbf{r})$ are real functions.

Let us first consider a non-degenerate eigenmode of a single cavity, which satisfies the equation

$$\widehat{\Theta}_c|F_c\rangle = \overline{\omega}_c|F_c\rangle, \quad (5)$$

where $\overline{\omega}_c$ is the eigenvalue of the Maxwell operator for the single cavity, $\widehat{\Theta}_c$. For a periodic array of identical cavities, centered at sites $\mathbf{R}_n$, we construct a basis of Bloch eigenfunctions, characterized by wavevectors $\mathbf{k}$, as follows [2]:

$$\mathbf{F}_\mathbf{k}(\mathbf{r}) = \sum_{n'}\exp(i\mathbf{k}\cdot\mathbf{R}_{n'})\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'}). \quad (6)$$

Then, substituting into the corresponding eigenvalue equation $\widehat{\Theta}|F_\mathbf{k}\rangle = \omega_\mathbf{k}|F_\mathbf{k}\rangle$, we can easily obtain

$$\sum_{n'}\exp(i\mathbf{k}\cdot\mathbf{R}_{n'})\Big[\int d^3r\,\mathbf{F}_c^*(\mathbf{r}-\mathbf{R}_n)\cdot(\widehat{\Theta}-\widehat{\Theta}_c)\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'}) + \overline{\omega}_c\int d^3r\,\mathbf{F}_c^*(\mathbf{r}-\mathbf{R}_n)\cdot\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'})\Big] = \sum_{n'}\exp(i\mathbf{k}\cdot\mathbf{R}_{n'})\omega_\mathbf{k}\int d^3r\mathbf{F}_c^*(\mathbf{r}-\mathbf{R}_n)\cdot\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'}). \quad (7)$$

In the spirit of tight-binding approximation, because of the localized nature of the cavity eigenmodes, we first assume that $\int d^3r\, \mathbf{F}_c^*(\mathbf{r}-\mathbf{R}_n)\cdot\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'}) \approx 4U_c\delta_{nn'}$, where $U_c = 1/4\int d^3r\, |\mathbf{F}_c(\mathbf{r})|^2$ is precisely the time-averaged EM energy of the single-cavity mode. Moreover, in the case of a linear chain of cavities centered at sites $\mathbf{R}_n = (0, 0, na)$, neglecting the on-site matrix element of $\widehat{\Theta} - \widehat{\Theta}_c$ ($\widehat{\Theta}$ and $\widehat{\Theta}_c$ differ only outside the given cavity) and considering only the intersite elements between nearest neighbors that we define to be equal to $2U_cW$, equation (7) leads directly, without the need of any additional approximation [4], to the first-order dispersion relation

$$\omega_k = \overline{\omega}_c + W\cos(ka), \quad (8)$$

which is identical to equation (2) for a single band.

An advantage of the present approach is that we can also derive a first-order linear differential equation for the time-evolution of the EM field in the cavities. A pulse that can be decomposed into Bloch eigenmodes of the waveguide, i.e. $\mathbf{F}(\mathbf{r}, t) = \sum_k c_k\mathbf{F}_k(\mathbf{r})\exp(-i\omega_k t)$, can be written, using equation (6), as

$$\mathbf{F}(\mathbf{r}, t) = \sum_{n'} A_{n'}(t)\mathbf{F}_c(\mathbf{r}-\mathbf{R}_{n'}), \quad (9)$$

where

$$A_{n'}(t) = \sum_k c_k \exp[i(kn'a - \omega_k t)]. \quad (10)$$

On the other hand, $\mathbf{F}(\mathbf{r}, t)$, given by equation (9), satisfies equation (3), i.e. $\widehat{\Theta}\mathbf{F}(\mathbf{r}, t) = i\partial_t\mathbf{F}(\mathbf{r}, t)$, and, following similar steps to those leading to equation (8), we obtain

$$i\frac{dA_n(t)}{dt} = \overline{\omega}_c A_n(t) + \frac{W}{2}[A_{n+1}(t) + A_{n-1}(t)]. \quad (11)$$

Equation (11), which describes the time evolution of a pulse propagating along the cavity-plasmon waveguide, has also been obtained by others [7, 8]. However, our approach provides a framework for a consistent and straightforward derivation of both equations (8) and (11). The coefficients $A_n(t)$ have a clear physical meaning. Since the energy of the pulse at time t is $\frac{1}{2}\int d^3r|\mathbf{F}(\mathbf{r}, t)|^2 = 2U_c\sum_n |A_n(t)|^2$, $|A_n(t)|^2$ give the energy distribution of the pulse over the different cavities at the given time. Using equation (8) and the mathematical identity

$$\exp(-iz\cos\phi) = \sum_{n=-\infty}^{\infty}(-i)^n J_n(z)\exp(in\phi), \quad (12)$$

where $J_n(z) = (-1)^n J_{-n}(z)$ are the Bessel functions [9], the general solution (equation (10)) of equation (11) takes the alternative form

$$A_n(t) = \exp(-i\overline{\omega}_c t)\sum_k\sum_{m=-\infty}^{\infty} c_k\exp[ik(n+m)a] \times (-i)^m J_m(Wt). \quad (13)$$

Subject to the initial conditions $A_n(t=0) = \delta_{n0}$, the above solution becomes

$$A_n(t) = (-i)^n\exp(-i\overline{\omega}_c t)J_n(Wt), \quad (14)$$

as can also be verified by direct substitution into equation (11) and using the recurrence relation $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$ [9]. Considering the solution for a non-dissipative system given by equation (14), it is straightforward to show that the mathematical identity $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$ [9] ensures energy conservation. As discussed in relation to figure 2, losses can be taken into account by adding a (constant) negative imaginary part, $-i\overline{\gamma}_c$, to the dispersion relation of the cavity-plasmon waveguide and thus, in this case, equation (14) becomes $A_n(t) = (-i)^n\exp(-i\overline{\omega}_c t - \overline{\gamma}_c t)J_n(Wt)$.

We now assume a finite chain of cavities $n = 1, 2, \ldots, N$. The eigenmode of a given cavity, say that at $\mathbf{R}_p$, is continuously excited by a harmonic light source of angular frequency ω_{ext}. In this case, equation (3) takes the form $\widehat{\Theta}\mathbf{F}(\mathbf{r}, t) = i\partial_t\mathbf{F}(\mathbf{r}, t) + b\omega_{ext}\exp(-i\omega_{ext}t)\mathbf{F}_c(\mathbf{r}-\mathbf{R}_p)$, where b is a coupling constant, which leads to the time-dependent equation

$$i\frac{dA_n(t)}{dt} = (\overline{\omega}_c - i\overline{\gamma}_c)A_n(t) + \frac{W}{2}[A_{n+1}(t) + A_{n-1}(t)] - b\omega_{ext}\exp(-i\omega_{ext}t)\delta_{np}, \quad (15)$$

where the quantity $-i\overline{\gamma}_c$ represents the losses, as discussed above. We seek a particular solution of equation (15) of the form $a_n\exp(-i\omega_{ext}t)$ and obtain a_n by solving the resulting linear system $\mathbf{Ma} = \mathbf{b}$, where $\mathbf{M}$ is a tri-diagonal $N\times N$ ($N\to\infty$) matrix with elements $M_{nn'} = (\overline{\omega}_c - \omega_{ext} - i\overline{\gamma}_c)\delta_{nn'} + \frac{W}{2}\delta_{nn'\pm1}$ and $b_n = b\omega_{ext}\delta_{np}$. It can readily be shown that $\mathbf{u}^{(k)}$, with $u_n^{(k)} = \exp(ikna)/\sqrt{N}$, where k takes the values $2\pi\nu/(Na)$, $\nu = 0, 1, \ldots, N-1$ is the complete and orthonormal set of eigenvectors of $\mathbf{M}$ (cyclic boundary conditions) and $\lambda^{(k)} = \overline{\omega}_c - \omega_{ext} - i\overline{\gamma}_c + W\cos(ka)$ is the corresponding set of eigenvalues. Therefore, since the inverse of $\mathbf{M}$ is $[\mathbf{M}^{-1}]_{nn'} = \sum_k u_n^{(k)}u_{n'}^{(k)*}/\lambda^{(k)}$, the particular solution of equation (15) becomes $(b\omega_{ext}/N)\exp(-i\omega_{ext}t)\sum_k\exp[ik(n-p)a]/[\overline{\omega}_c - \omega_{ext} - i\overline{\gamma}_c + W\cos(ka)]$ and, considering that the general solution of the corresponding homogeneous equation (see equation (10)) is $A_n(t) = \sum_k c_k\exp\{i[kna - (\overline{\omega}_c - i\overline{\gamma}_c)t - Wt\cos(ka)]\}$, we obtain the general solution of equation (15):

$$A_n(t) = \exp(-i\overline{\omega}_c t - \overline{\gamma}_c t)\sum_k c_k\exp\{i[kna - Wt\cos(ka)]\} + \frac{b\omega_{ext}\exp(-i\omega_{ext}t)}{N}\sum_k\frac{\exp[ik(n-p)a]}{\overline{\omega}_c - \omega_{ext} - i\overline{\gamma}_c + W\cos(ka)}. \quad (16)$$

Let us now consider the above (general) solution subject to the initial conditions $A_n(t=0) = 0$. For an infinite number of cavities ($N\to\infty$), the sums over k can be transformed into contour integrals over the unit circle and, using equation (12), we finally obtain

$$A_n(t) = \frac{2b\omega_{ext}}{W(z_< - z_>)}\Bigg\{\exp(-i\omega_{ext}t)z_<^{|n-p|} - \exp(-i\overline{\omega}_c t - \overline{\gamma}_c t)\Bigg[\sum_{m=-n+p+1}^{\infty}(-i)^m J_m(Wt)z_<^{n-p+m}$$

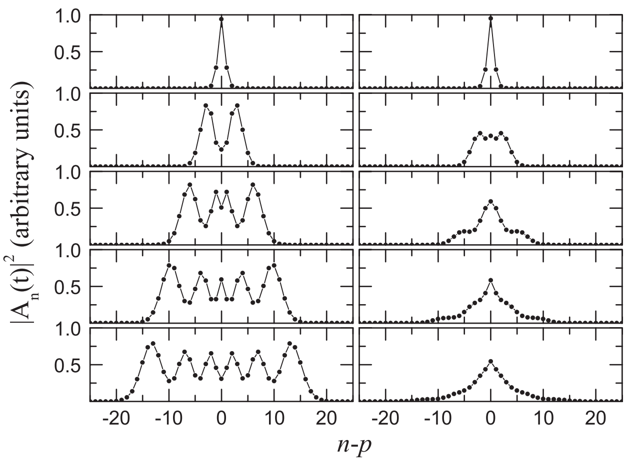

Figure 3. Snapshots showing the time evolution, from $t = 0.1$ ps (top) to $t = 0.9$ ps (bottom) with a step of 0.2 ps, of the EM energy distribution over the coupled-cavity waveguide of figure 2. The waveguide is excited by a monochromatic light source with a frequency at the center of the single band ($\hbar\overline{\omega}_c = 2.508$ eV). Left-hand diagram: full compensation of losses ($\hbar\overline{\gamma}_c = 0$). Right-hand diagram: incomplete compensation of losses ($\hbar\overline{\gamma}_c = 4$ meV).

$$+ \sum_{m=n-p+1}^{\infty} (-\mathrm{i})^m J_m(Wt) z_<^{-n+p+m} + (-\mathrm{i})^{n-p} J_{n-p}(Wt) \Bigg]\Bigg\}, \qquad (17)$$

where $z_<$ and $z_>$ are the two roots of the second-order equation $z^2 + 2z(\overline{\omega}_c - \omega_{ext} - \mathrm{i}\overline{\gamma}_c)/W + 1 = 0$, with $|z_<| < 1$ and $|z_>| > 1$, respectively.

In figure 3 we show the time evolution, from $t = 0.1$ to 0.9 ps, of the EM energy distribution over the coupled-cavity waveguide of figure 2, given by equation (17). The waveguide is excited by a monochromatic light source with a frequency of $\hbar\overline{\omega}_c = 2.508$ eV, the eigenfrequency of the non-degenerate mode of the single spheroidal cavity. In the lossless case ($\hbar\overline{\gamma}_c = 0$) we see that the field propagates without attenuation in the two directions of the chain. If losses are present ($\hbar\overline{\gamma}_c = 4$ meV) the field fades out and becomes vanishingly small after about ten lattice constants. Moreover, the temporal oscillations of the energy, which are characteristic of the propagating case, are smoothed out after about 0.9 ps and a steady-state energy distribution is restored in the system. The above results indicate that, in order to achieve long-range propagation, absorptive losses in the system should be overcome.

Acknowledgments

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